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ABSTRACT

Forty kindergarten children at the Stephen Bull School in Racine, Wisconsin were tested to determine the effects of a sequence of 12 experimental lessons on the ability of kindergarten children to recognize and conserve numerousness. Subjects were 40-low-to-middle socioeconomic level children divided into treatment and control groups. A specially developed test of numerousness (arithmetic readiness) served as a pre- and posttest. The lessons were designed to give experience with one-to-one correspondence and comparisons by counting, relative size and/or relative density. No significant differences were observed between the mean gain scores of the experimental and control groups. However, significant differences were observed between the mean gain scores of children in the treatment groups who attended the half-day session and those attending a special full-day program. A similar result was observed within the control groups. Results indicate that the lessons used in this experiment did not alone sufficiently enhance the subjects' ability to conserve numerousness, but that they should provide an effective supplement to formal activity with number concepts. (Author/WY)

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Working Paper No. 44

THE EFFECTS OF INSTRUCTION ON THE DEVELOPMENT OF THE CONCEPT OF CONSERVATION OF NUMEROUSNESS BY KINDERGARTEN CHILDREN

by

Thomas L. Benzinger

Report from the Project on
Individually Guided Elementary Mathematics
Phase 2: Analysis of Mathematics Instruction
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The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to generate new knowledge about the conditions and processes of learning and about the processes of instruction, and the subsequent development of research-based instructional materals, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Working Paper is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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I

INTRODUCTION

In recent years the concern of Jean Piaget for conservation and its relation to mathematical experience for children has been the subject of many research projects. This paper is concerned with the problem of the attainment of conservation.

Piaget, in his book, The Child's Concept of Number, hypothesizes that ". . . in each and every case, the conservation of something is postulated as a necessary condition of any mathematical understanding" (Piaget, 1952, p. 3-4). He also demonstrates the existence of three distinct stages in the development of this concept. In general terms, a child is said to exhibit the property of conservation of numerousness if he conceives that the number of objects in a set is the same regardless of how the objects are physically rearranged. Thus a child will possess this property if he can ignore all other properties of a set, such as density of the set, the area that the set occupies, the color of the objects, etc., and note only the number of objects.

In this study an attempt is made to induce conservation of numerousness by exposing children presumed to be slightly below the age of onset of conservation to systematic learning experiences

designed to develop and bring into play factors believed to be important in the development of conservation. Any significant change in the children's performance indicating a tendency to possess conservation of numerousness should reflect the role played by the particular factors involved. At the same time the detailed examination of the interrelation among tasks involving conservation of numerousness should serve to extend our understanding of the nature of this problem.

The theoretical background for the study is imbedded in the developmental psychology of Jean Piaget (Flavell, 1963). His studies of child development and activity have produced an abundance of ideas on how children learn and think. The mathematical backdrop for the psychology involved is based on Set Theory and the concept of a mapping of one set into another. It deals with the numerical properties of sets of objects.

A set can be described as a collection of objects of any kind whatever. It may be thought of as being formed by the grouping together of single objects into a whole.

If a set A is a finite set, them $\overline{A}$ denotes the number of elements in A. $\overline{A}$ is called the cardinal number of the set A. Therefore, for any natural number $n > 0$, $\overline{A} = n$ if A is any set containing n elements and $\overline{A} = 0$ if A is an empty set.

"A mapping φ from a set A to a set B is one to one if for a_1, a_2 εA, than $a_1 \neq a_2$ implies that $\varphi(a_1) \neq \varphi(a_2)$. The mapping φ is onto if for every $b \varepsilon B$ there exists an $a \varepsilon A$ such that $\varphi(a) = b$" (Herstein, 1965, p. 12). There exists a one-to-one correspondence between the sets A and B if there exists a mapping φ from A to B that is both one-to-one and onto. That is, there exists a one-to-one correspondence between the sets A and B if we can establish a pairing of the elements of the set A with the elements of the set B in such a way that each element of A corresponds to one and only one element of B, and each element of B corresponds to one and only one element of A.

Two sets A and B are said to be equivalent if they can be placed in one-to-one correspondence. Equivalent sets are necessarily in the same equivalence class and therefore have the same cardinal number. Equivalent sets are equal if their members are identical. We say that the set A is less than the set B (or $\bar{A} < \bar{B}$) if there exists an onto mapping from B to A but no onto mapping from A to B.

It is possible to find the cardinal number of a finite set by a process of rational counting and it is known that young children have difficulty with this counting process. They may obtain the correct cardinal number of a set by a counting process but still have little notion of what they have done. Counting _per se_ is no guarantee that a child grasps what the concept of cardinal number is, or how it applies to a concrete situation. The child must also realize that no matter how he counts the elements of a set, the cardinal number of the set will always be the same.

Piaget has shown that young children may have great difficulty in maintaining the equality of the cardinal numbers of two sets when the correspondence has been changed for the child by an alteration of the elements or by an alteration of the order (Piaget, 1952). To understand what a number is, the child must be able to manipulate and make judgments about perceived objects in such a way that the order, or perceived pattern of elements in a set of objects, does not influence judgments about the number of objects present. Piaget lists as necessary conditions for understanding numbers: (1) the ability to deal with equivalence of cardinal classes in terms of one-to-one correspondence; and (2) the ability to deal with transitive relations such as "more than" and "less than."

There are several different theories on how a child builds his earliest notions of number (Davis, 1967). Among these are the following:

1. By studying sets, and the various attributes of sets including the numerousness of the objects in the collection.

2. By getting first the idea of "more," "less," and equality, and therefore giving number names to "as many as I have fingers," etc.

3. By studying invariance, as in the fact that rearranging pebbles in a different pattern or order does not change the number of pebbles present.

4. By experience in performing the act of counting, which is regarded as a human act that is learned by imitation, much as a child learns to swing a baseball bat (Davis, 1967, p. 21).

Concepts and methods suggested in these theories will be employed in our attempt to induce conservation of numerousness. Of the theories noted above, the last seems to provide the least satisfactory approach. This view is evidenced in a study by Wheatly during which he administered an individual test of conservation (number and length), cardination, and counting to first grade children during the first month of school and again in the last month of school (Wheatly, 1968).

The correlations of total test scores and subtest scores with the end-of-the-year achievement scores were found to be highly significant between scores of conservers and non-conservers. However, when the sample was dichotomized on the basis of counting ability, no significant difference in achievement scores of the two groups resulted. This finding also substantiates Piaget's theory on the importance of conservation in developing a stable concept of number.

In a recent study involving number training techniques for children, Egan Mermelstein (1968) states:

> The advocates of early intervention programs, such as Program Head Start, maintain that such programs improve the child's intellectual development. They suggest that this intellectual development will result in a corresponding improvement in academic achievement and school success. More specifically, they contend that the acceleration of number development, a dimension of intellectual development, has obvious implications for success in later number work. In support of this contention, Almy, et al. (1966), have demonstrated that children who conserve at an early age do better in beginning reading and arithmetic than those who are non-conservers (p. 1).

In Piaget's theory of intellectual development a central role is assigned to the child's conceptualization of the principle that a particular dimension of an object may remain invariant under changes involving other irrelevant aspects of the situation. For instance, children who lack conservation think that equivalence between two sets of objects no longer holds following a change in the arrangement of the elements of one set or both. Piaget offers the following example:

> A child of five or six may readily be taught by his parents to name the numbers from 1 to 10. If 10 stones are laid in a row, he can count them correctly. But if the stones are rearranged in a more complex pattern or piled up, he no longer can count them with consistent accuracy. Although the child knows the names of the numbers he has not yet grasped the essential idea of number; namely, that the number of objects in the group remains the same, is 'conserved', no matter how they are shuffled or arranged (Piaget, 1953, p. 76).

To Piaget conservation is a necessary condition for all rational activity. Assuming this as a postulate, arithmetical thought is no exception to the rule. In fact, the domain of the number concept lends itself particularly well to the investigation of the development of conservation for several reasons:

1. The number dimension occupies a unique position in regard to the question of conservation insofar as the cardinality of a finite collection is an exact measure.

2. In this domain the problem of conservation can be readily related to development in other aspects of the number concept rather than constituting a somewhat isolated problem.

3. Recent empirical work by Dodwell (1960), Elkind (1961), and Van Engen and Steffe (1966) has given strong support to the notion that the attainment of the level of conservation marks a clearly defined stage in the formation of the number concept.

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In Piaget's theory a finite set is only conceivable if it remains unchanged, irrespective of any changes occurring in the relationships between its elements. But further, according to Piaget, whether it be a matter of sets and number conceived by thought or of the most refined axiomatization of an intuitive system, in each and every case, conservation of something is postulated as a necessary condition for any mathematical understanding.

In particular, conservation of numerousness means that irrespective of how a set of objects is rearranged, the number of objects remains the same. In other words, the cardinal number of a set is independent of the arrangement of its members. In _The Child's Concept of Number_, Piaget (1952) demonstrates the following stages in the development of this concept:

1. _Absence of Conservation_. A child is totally unable to ignore his perceptions. He may be misled by a comparison of relative sizes or by the relative density of objects, when attempting to judge the sameness of number.

2. _Necessary Conservation_. A child is able to ignore his perception. He recognizes the number of objects in a set to be the same regardless of the arrangement of rearrangement of the objects.

3. _Intermediary Reactions_. This is a transition stage from the absence of conservation to the necessary conservation stage. The child is not consistent in his response. In separate cases, the response may fit either of the above stages (pp. 5-13).

Piaget says this of these stages: "The ordering is constant, and has been found in all societies studied. However, although the order of succession is constant, the chronological ages of these stages vary a great deal. Typically, conservation of numerousness is found with greater frequency among older children; that is,

children in a more advanced stage of mental growth."

In his writings, Piaget has stated that the development of the intellectual capacity of children depends on at least the following four main factors: (1) maturation, (2) experience, (3) social transmission, and (4) equilibration (i.e., a process of self-regulation or a development of logical structures when confronted with cognitive conflict). Equilibration is an active process which leads to reversibility. It is a more fundamental factor in development than the other three. In the conservation problem one can always find a process of self-regulation which Piaget identifies as the fundamental factor in the acquisition of logical-mathematical knowledge (Ripple & Rockcastle, Eds., 1964, p. 10).

Piaget also identifies three types of quantitative comparisons which are observable in children as:

1) Gross quantity - perceived relations (longer than, larger than) between objects which are not coordinated with each other. That is, comparisons of the type "more" or "less" contained in judgments such as "it's higher," "not so wide," etc.

2) Intensive quantity - any magnitude which is not susceptible of actual addition. An example is temperature.

3) Extensive quantity - any magnitude that is susceptible of actual addition. An example is mass or capacity (p.5).

Regarding these types of quantity, Piaget says, "The child does not first acquire the notion of quantity and then attribute constancy to it. The question to be considered is whether the development

of the notion of conservation of quantity is not one and the same as the development of the notion of quantity."

Piaget suggests reversibility as a possible factor on which the attainment of conservation might crucially depend. It is clear that experience of some kind is also involved, but it is not easy to specify just what this experience is. Wallach and Sprott (1964) extend Piaget's view of the role of reversibility as they urge that reversibility is the prime mechanism of conservation. They attempt to induce conservation of number by showing children the reversibility of rearrangements. That is, leading the child to realize that rearranged objects which fit together before rearrangement can be made to fit together again. The conjecture is that conservation is caused by actually thinking of an inverse operation and realizing that it would bring about again the original situation. Thus, conservation can result from the recognition of reversibility, although reversibility may be known without conservation.

However, Elkind believes that this knowledge of reversibility is of little value if the child is not already convinced of conservation (Elkind, 1967). He views conservation as being attained through the utilization of a deductive argument. The employment of verbal explanations such as reversibility (if you make it like it was before, it will be the same), identity (nothing has been added or taken away, so it is the same), and compensation (what is lost in one way is gained in another) merely reflect the attempt to give a logical explanation to the conservation judgment.

The significance of these verbal explanations lies in that they imply that the child now feels that conservation is a logical necessity and that he must justify it. Therefore, according to Elkind (1967), "conservation involves deduction, and verbal explanations are really post hoc rationalizations rather than veridical reflections of the process leading to conservation. Their only value is that they illustrate the child's new operational or logical orientation. If the child were really to verbalize the way in which he arrived at the solution, he would say something like this: This set (A) had the same number as that set (B) before, and the change ($A \rightarrow A'$) doesn't change anything, so this set (A') must still have the same number as that set (B'). Then it would appear that conservation is not in itself a numerical notion, but rather it is a logical concept" (pp. 17-18).

From this example of conflicting opinions it seems rather apparent that the difficult task of reading Piaget causes his views to be widely misinterpreted and misunderstood. At this point, it does not appear that anyone has identified with a sufficient degree of certainty the factors responsible for inducing number conservation in children. However, Dodwell (1960) conducted studies designed to replicate some of Piaget's tests of conservation and found that development of conservation proceeded in much the same fashion as indicated by Piaget. Ina Uzgiris (1964) replicated some of Piaget's work to verify that a child's reasoning becomes operational in mathematical and logical operations at about the age of 7 years.

Piaget is not clear on his stand on the issue of inducing conservation of numerousness through teaching. At one time he indicates that teaching could have an important effect on a child's ability to conserve number (Piaget, 1964, in Ripple and Rockcastle). However, on another occasion his statements seem to imply that teaching is not an important factor in the acquisition of conservation of numerousness but rather that the child develops the notion of number independently and spontaneously, and true understanding comes only with his mental growth (Piaget, 1953).

Piaget is also vague on the matter of enhancing the learning of conservation at an early age. However, his statement that, "children must grasp the principle of conservation of quantity before they can develop the concept of number," certainly illustrates his beliefs concerning the importance of children being able to conserve both number and substance.

Independent studies by LeBlanc (1968) and Steffe (1966) have shown that there exists a high relationship between a test involving conservation of numerousness and problem solving involving subtraction and addition in arithmetic. They conclude that conservation of numerousness is at least a necessary condition for success in problem solving. It follows from this that an improvement in a conservation test score might indicate an improvement in conservation ability, which in turn could imply a greater probability for success in arithmetic.

Since the concept of conservation has been acknowledged as the precursor to number ability, researchers have investigated the feasibility of inducing the concept through training or experience. The literature on this topic appears to be rather equivocal. Wohlwill and Lowe (1962) have been unsuccessful in their attempts to induce conservation of numerousness. Other studies by Smedslund (1961), Prager (1966), and Mermelstein, et al. (1966), suggest that regardless of the kind of conservation that one tries to induce with diverse populations, in general, such training is not successful.

However, on the other hand, Wallach and Sprott (1964) have successfully induced number conservation through reversibility training. Other investigators such as Churchill (1958), Gruen (1965), Roeper and Sigel (1965), and Beilin (1965) have claimed success in inducing various kinds of conservation by a variety of training techniques. It should be noted that the studies which claim success in inducing conservation all utilized middle-class populations in contrast to the wide range of populations utilized by the researchers in the conservation studies described as unsuccessful.

Therefore, in view of Piaget's hypothesis that the conservation of something is postulated as a necessary condition of any mathematical understanding, and the evidence presented in recent studies which support theoretical reasons for believing that the concept of conservation of numerousness is a prerequisite for

problem solving in arithmetic, the question first arises as to whether or not children can be taught the concept of conservation of numerousness and secondly, whether activities responsible for improving the notion of conservation of numerousness can be identified.

This study was initiated to attempt to answer these questions by examining whether the conservation of numerousness ability of kindergarten children as measured by a specific pre-test can be improved as a result of administering special lessons that are based on the concept of one-to-one correspondence.

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II

METHOD

DESIGN

A group of 40 subjects was available for the experiment. They are members of the kindergarten class at Stephen Bull School in the Unified School District No. 1 of Racine, Wisconsin. The population is made of low to middle socioeconomic level children, and include 20 former Head Start pupils. Some of the children participated in the school's Developmental Full-day Program, an experimental follow-up to Program Head Start that is sponsored by the local school district.

The population was randomly divided into a treatment group and a control group, each containing 20 children. The treatment group received special conservation-oriented instruction, while the control group proceeded with normal arithmetic activity. This consisted of materials, games, and activities for the most part related to counting. There is no formal instruction in arithmetic at the kindergarten level. After the original division of the population into a treatment and control group and the instructions had begun, it was decided that it would be interesting to investigate the possibility of differences between the full-day children

and those not in the Developmental full-day program within the treatment and control groups.

Children involved in the Developmental full-day program attend school during both morning and afternoon sessions, as opposed to a normal half-day routine for the other children. As part of the afternoon program, the full-day children received a daily 20-minute period of S.R.A.-orientated instruction in number concepts.

Therefore, another division of the population was necessary. Since the treatment had already begun, the only solution was to subdivide each of the treatment and control groups into full-day and half-day sections. This second division formed the following four groups, for analysis purposes:

1. Treatment and full-day (7 children)
2. Treatment and normal half-day (13 children)
3. Control and full-day (5 children)
4. Control and normal half-day (15 children)

Obviously, these selections are not completely random. Instead, the four groups were formed by means of a pseudo-random technique, thus establishing the unequal cell frequencies. These unequal cell n's do however reflect more accurately the true ratio of full-day children to kindergarten children not participating in the Developmental Full-day Program. The experimental design is presented in Figure 1.

Diagram of Design

Experimental Groups		Control Groups		
g_1	g_2	g_3	g_4	
$x_{1,1}$	$x_{1,2}$	$x_{1,3}$	$x_{1,4}$	
.	.	.	.	
.	.	.	.	
.	.	.	.	
$x_{7,1}$	.	$x_{5,3}$	.	
	$x_{13,2}$		.	
			.	
			$x_{15,4}$	
$x_{\cdot 1}$	$x_{\cdot 2}$	$x_{\cdot 3}$	$x_{\cdot 4}$	$x_{\cdot\cdot}$
lessons and full-day	lessons and half-day	normal and full-day	normal and half-day	

Figure 1: Experimental Design

		Full-day	Half-day
Treatment	Experimental	7	13
	Control	5	15

In this study, the interest is not solely in the overall existence of treatment effects, but rather from the outset our primary interest lay in examining the following null hypotheses.

Hypothesis 1. There is no difference in the mean scores observed between the treatment and control groups.

Hypothesis 2. There is no difference in the mean scores observed between the treatment group attending the half-day session and the treatment group in the Developmental Full-Day Program.

Hypothesis 3. There is no difference in the mean scores observed between the control group attending the half-day session and the control group in the Developmental Full-day Program.

In order to examine the plausibility of these hypotheses a technique of planned comparisons among means is applicable. The planned comparison procedure outlined in Hays (1966) was used for this purpose.

In examining Hypothesis 1, a pretest-posttest control group design was used. The subjects were randomly assigned to the experimental or control group from a common population. The control group engaged in normal arithmetic activity during the period corresponding to that in which the experimental group received the treatment.

The ability of this design to control sources of internal and external invalidity is shown in Table 1, wherein plus (+) indicates that the factor is controlled, a minus (-) indicates a definite weakness, and a question mark (?) indicates a possible source of concern.

Table 1

INTERNAL AND EXTERNAL INVALIDITY CONTROLS

Internal	History	+
	Maturation	+
	Testing	+
	Instrumentation	+
	Regression	+
	Selection	+
	Mortality	+
	Interaction of selection and maturation, etc.	+
External	Interaction of testing and treatment	-
	Interaction of selection and treatment	?
	Reactive arrangements	?

(from Campbell & Stanley, 1963, p. 8)

Since the factors of internal invalidity directly affect pretest and posttest scores they could produce changes which may be mistaken for the result of the treatment. Obviously, from the chart, these factors are almost completely controlled.

Therefore, any problem incurred in this design would be one of external validity. However, these external validity

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problems were solved by taking appropriate precautions to avoid attitudes that would be unrepresentative of the normal school setting.

A nonequivalent control group design proposed by Campbell and Stanley (1963) was used to examine Hypothesis 2 and Hypothesis 3. This quasi-experimental design is appropriate since the two groups involved "do not have pre-experimental sampling equivalence but rather constitute naturally assembled collectives" (Campbell & Stanley, 1963, p. 47). Here the assignment of the treatment (Full-day Program) is assumed to be random and under the experimenter's control.

The internal validity controls of this design are not as extensive as those of the first, and the threat of testing to external validity remains. However, it is certainly adequate for this phase of the experiment.

DATA COLLECTED

The data collected for reach of the subjects included a pretest score, a posttest score, I.Q., and child's chronological age. The mean I.Q. of the population was 96.8. Their chronological ages ranged from 4 years 9 months to 6 years 6 months with an average age of 5 years 3 months.

The Test of Conservation of Numerousness which was used as the pretest in this study is an arithmetic readiness test for kindergarten and first grade. It was developed in a two-year period

1965-1967 at the University of Wisconsin by E. Harold Harper and Leslie P. Steffe under the direction of Professor Henry Van Engen. The original effort produced an individual test which used 3-dimensional objects as items. In this individual test the subject did not perform any manipulation of objects. All actions were carried out by the examiner.

Characteristics of the Test of Conservation of Numerousness were studied utilizing subjects from Oconomowoc, Wisconsin (Steffe & Harper, 1968, p. 23-32). The subjects were a group of 124 randomly selected kindergarten children within the age range of 5 years 2 months and 6 years 4 months. Their mean I.Q. was 105. The internal consistency reliability of this test of conservation of numerousness was .87.

Subsequently the individual test was modified so that it could be administered to small groups. It is this latter group instrument that was utilized in the present study. The relationship of the group test to the individual test was investigated in a study utilizing children at Cottage Grove, Wisconsin. A correlation between the two forms of .84 was obtained when children were initially tested on the group form, and one month later tested with the individual form of the test.

Both tests require about 40 minutes of testing time for administration of the four warm-up and 16 test questions. The warm-up items for the test are essentially training items. They involve either quantitative comparisons or such a small number of

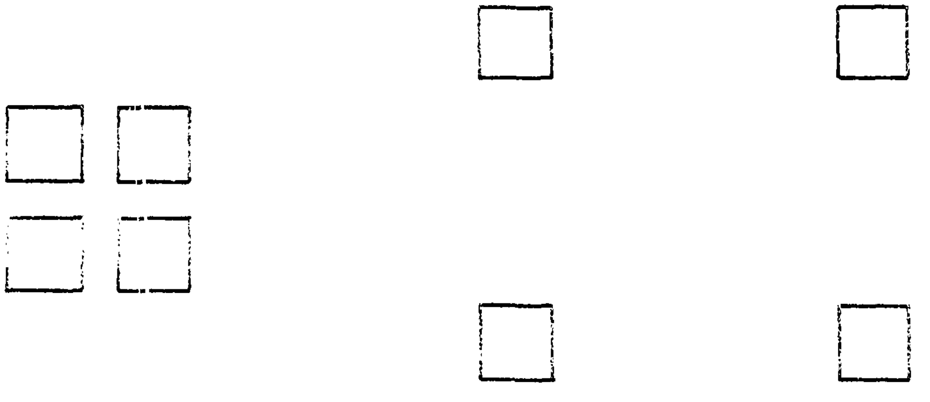
objects that the children would have a very easy time answering them. Eight of the test items involve objects that are static and eight of the items involve movable objects. Of the eight items involving static objects, six involve comparison of two equal sets. The largest cardinal number of all the sets to be examined by the subject was eight. The geometrical configurations varied among circles, rectangles, lines, and triangles.

The remaining eight items of the test involve objects which the child moves. These items presented situations in which the child had to compare two sets of objects after a rearrangement of the elements of one of the sets. Here the one-to-one correspondence is actually established by the children before they are asked to compare the two sets in their final state.

It was possible for the children to make responses to the items by using the following methods: (1) one-to-one correspondence, (2) comparison by counting, (3) comparison by relative sizes, (4) comparison by relative density, or (5) no comparison (guessing). The first and second methods involve comparisons of extensive quantity. The third and fourth methods are based on comparisons of gross quantity. Intensive quantity might be illustrated as a combination of the third and fourth methods.

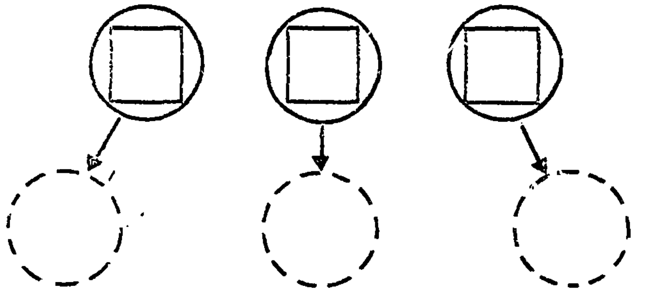
The materials used in the testing were test booklets and small cardboard discs. Each child performed his own manipulations according to the directions given by the tester. The directions accompanying the Test of Conservation of Numerousness were found to be too complex for the kindergarten children, and therefore were modified to comply with their vocabulary and level of understanding. The following are examples of the four basic problem types appearing on the Test of Conservation of Numerousness, along with the instructions issued by the tester.

EXAMPLE 1



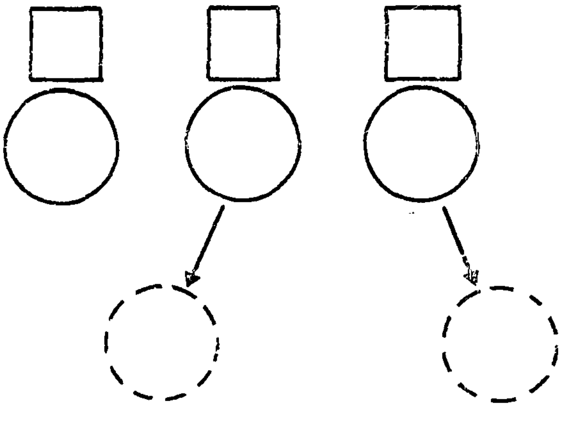
INSTRUCTION: Look at the squares on both pages. If you think there are the same number of squares on each page, point to both pages by placing one hand on each page. But if you think there are more squares on one page than the other, point only to the one that is more. Show me.

EXAMPLE 2



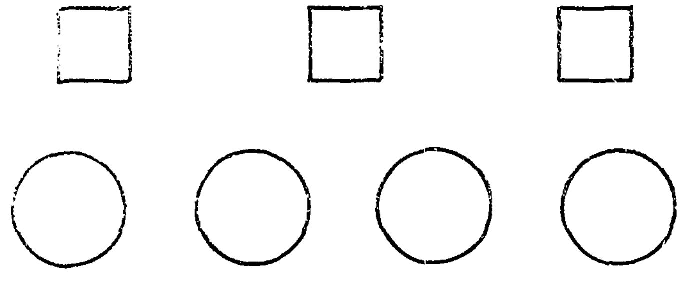
INSTRUCTION: Use three discs. Cover each square with a circle. Move one circle to cover each dot. If you think there are the same number of squares as circles point with one hand to the squares and with one hand to the circles. But if you think there are more of one than the other, point only to the one that is more. Show me.

EXAMPLE 3



INSTRUCTION: Use three discs. Place a circle next to each square. (So we can see both the circles and squares.) Move a circle to cover each dot. If you think there are the same number of squares as circles point with one hand to the circles. But, if you think there are more of one than the other, point only to the one that is more. Show me.

EXAMPLE 4



INSTRUCTION: Use four discs. Cover each dot with a circle. If you think there are the same number of squares as circles point with one hand to the squares and with the other hand to the circles. But, if you think there are more of one than the other, point only to the one that is more. Show me.

TREATMENT

After completion of the pretesting in which all 40 children were tested 3 at a time, by an experienced teacher and tester, the treatment group received 12 30-minute sessions of special instructions, presented by a teacher and an assistant. During these instruction periods the control group continued with normal kindergarten activities. The special lessons were developed by Leslie Steffe and E. Harold Harper at the University of Wisconsin Research and Development Center for Cognitive Learning during the period from 1965-1967 (Steffe & Harper, 1968). They grew out of the analysis of a pilot study and the authors' personal experiences in teaching primary children and supervising teachers of primary children.

These lessons were designed to develop and give experience with factors believed to be important for the attainment of conservation of numerousness in children. They were prepared in a manner that would allow the activities to progress from physical activity involving all the children, to concrete manipulation of physical objects by the children, to semiconcrete illustrations by the teacher on a flannel board.

The objectives of the lessons are:

1. To demonstrate that, by matching or pairing, two sets can be put in one to one correspondence.

2. To develop the child's ability to judge the equivalence of two sets.

3. To understand the meaning of the phrases, "as many as", "more than", and "fewer than."

4. To demonstrate the constancy of numerousness during the movement of sets and rearrangement of elements.

5. To develop the ability to construct sets having the same number of elements.

6. To emphasize constancy of number in operations involving transformations related to addition and subtraction.

7. To provide experience with one to one correspondence and instances involving the concepts of "as many as", "more than" and "fewer than."

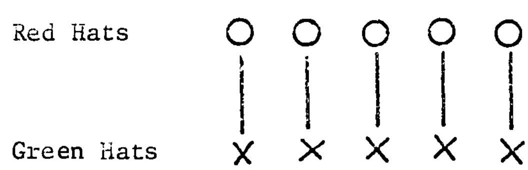
The activities are varied to capitalize on the use of concrete experiences and to present the child with situations in which he himself experiments. In some activities the teacher presents demonstrations with cutouts at a flannel board. This method is used to exhibit the existence of a one-to-one correspondence between two sets and to give the children an opportunity to observe the effect

of rearrangement of the elements in one or both of the sets. However, since action is one of the bases of effective learning, the child also participates in physical action games designed to extend the ideas presented at the flannel board. This physical action is the foundation for the mental operation we wish to develop. The ultimate goal is to create opportunities for the child to be less and less dependent on physical action until the action is internalized as a mental operation.

Probably the most effective way to describe the nature of the lessons is to examine the following examples of the activities and games employed.

Example 1 (from Lesson 1)

Split the group into two equal sets of children. Give one set green paper hats and the other set red paper hats. Have the children line up in two lines to form pairs.



Ask, "Does everyone have a partner? Does each one with a red hat have a partner with a green hat? Does each one with a green hat have a partner with a red hat?" Have the children with green hats change places. Ask the questions again. Change again and again, asking similar questions.

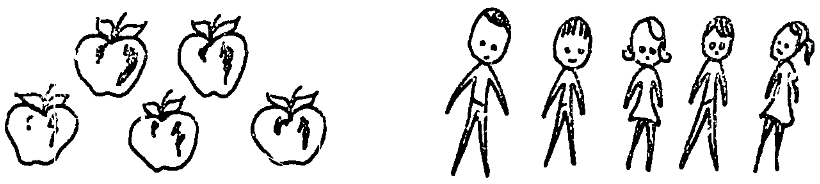
Example 2 (from Lesson 2)

Take two sets of felt figures (ducks and fish). Place one set on the felt board in a straight row. Ask a child to put the other in a row right under the row you put up. Then ask, "Is there a duck for each fish? Is there a fish for each duck?"--thus leading to the use of the term "as many as." Summarize this step by saying, "Each duck is paired with a fish, and each

fish is paired with a duck. We have matched the set of ducks with the set of fish. What can we say about the two sets?" Repeat using different arrangements of the elements of the sets.

Example 3 (from Lesson 5)

Use five apple cutouts and five children cutouts and place them on the flannel board in a pattern like that indicated below:



Then ask, "Is there an apple for each child? How could we find out?" If a child suggests matching or pairing the cutouts, let him do so as the group watches.

Example 4 (from Lesson 9)

Have 10 children form two lines in different places at the front of the room. Put six in one line and four in the other. Ask, "Which line has more than the other? Which line has fewer than the other? How can we tell?" [Have the 2 lines pair off to confirm their statements.] With the 2 lines next to each other ask, "How could we make this short line have "as many as" the other line?" [The children will no doubt suggest adding two more children to the short line or possibly subtracting two from the longer line. Tell them that we are not allowed to do this; instead, we must use only the ten children we have.] Lead them to see that by taking one from the long line and putting that person in the short line we make both lines the same length. That is, one will have as many as the other.

According to Piaget (1953) experiments with one-to-one correspondence are very useful for investigating children's development of the number concept. These instructions introduce the concept of one-to-one correspondence in the context of basic set theory. Through an intuitive set approach we strive to impress the children

with the fact that two sets can be put in one-to-one correspondence and that this fact can be used to judge the equivalence of the two sets.

As a result of these instructions, we want to develop the following reaction to the conservation problem: When it is apparent that two sets can be put in one-to-one correspondence (clearly matched), their continued sameness of number despite changes in arrangement should be recognized when it is recognized that they could be matched again. Then through experience with one-to-one correspondence we strive to promote the transfer of this reaction to new sets of objects, thus maintaining the conservation concept.

In some instances during the course of the experiment it was necessary to alter the activities to conform to the facilities and available materials at Bull School. In other cases the suggested activity was found to be ineffective and had to be modified or replaced by an improved routine. However, each concept suggested in the lessons was presented in some manner.

The Test of Conservation of Numerousness was administered a second time, in the role of posttest, following completion of the 12 instruction periods.

III

RESULTS

The frequency distribution of total scores for the pretest is given by Table 2 and the frequency distribution for the posttest scores is given by Table 3.

The complete data listings are given in Appendix A.

As is easily observed, there is some difference between the pre- and post-test distributions. The pretest distribution for all subjects tends toward the normal, while the post-test distribution for all subjects is negatively skewed. The treatment group shows a slightly higher mean increase than the control group. All of the groups have a higher mean score in Table 2 than in Table 1.

At least three factors could account for this gain. They are: (1) maturation; (2) familiarity with the test; and (3) experiences with number concepts. The first is not subject to experimental control and could only be evaluated in a situation which presented a total lack of familiarity with the test and no contact with experience involving number concepts. An attempt was made to control the second factor by allowing 13 weeks between testings. However, some retest familiarity may

Table 2

FREQUENCY DISTRIBUTION FOR PRETEST SCORES

Total Score	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Mean
Control Group	1	0	0	1	0	1	1	1	1	2	3	0	3	3	2	1	0	9.75
Treatment Group	0	0	1	0	3	0	0	5	1	0	3	0	3	4	0	0	0	8.75
All Subjects	1	0	1	1	3	1	1	6	2	2	6	0	6	7	2	1	0	9.25

Table 3

FREQUENCY DISTRIBUTION FOR POSTTEST SCORES

Total Score	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Mean
Control Group	0	1	0	0	0	0	0	1	3	1	2	1	5	2	4	0	0	10.70
Treatment Group	0	0	0	1	1	0	1	1	1	0	1	3	1	5	5	0	0	10.90
All Subjects	0	1	0	1	1	0	1	2	4	1	3	4	6	7	9	0	0	10.80

be present. The third factor is subject to experimental control and was controlled to the extent of allowing only normal classroom activities. These activities included informal emphasis on counting.

Four distinct groups were observed in this experiment (p. 16). The mean gain scores of the four groups were as follows:

Table 4

MEAN GAIN SCORES FOR GROUPS

Group	g_1	g_2	g_3	g_4
Mean	4.86	.69	4.20	-.13

For a more complete tabulation of individual gain scores see Appendix B. The mean square error found by the usual method for a one-way analysis of variance was 16.34 with 36 degrees of freedom. The three hypotheses to be examined are shown in Table 5 in the form of comparisons among means using an appropriate set of weights:

Table 5

WEIGHTS FOR COMPARISONS AMONG MEANS

Comparison	Means			
	g_1	g_2	g_3	g_4
1	1	1	-1	-1
2	1	-1	0	0
3	0	0	1	-1

The first hypothesis (H_o: $\psi_1 = 0$), i.e., there is no difference in the mean scores observed between the treatment and control groups, is supported by the data as evidence by the test statistic $t_1 = .524$. Therefore, we fail to reject the null hypothesis. However, of the 20 children in the treatment groups 15 showed a positive gain score. The mean gain for these children was 4.6. One child's score remained constant, and of the four children showing losses, the average loss was 6.5. At this time it is not possible to determine the cause of this large average loss in the treatment group. It is interesting, however, to note that the IQ range for this group is 77-105.

In the control groups, 12 children gained and 8 children showed a negative gain score. Here the mean gain was 3.3. while the 8 children showing a loss had a mean of 2.8. In this case, some of the loss can be attributed to a natural regression to the mean. But this would not explain the higher loss for the treatment groups.

The second hypothesis (H_o: $\psi_2 = 0$) i.e., there is no difference in the mean scores observed between the treatment group attending the half-day session and the treatment group in the Developmental Full-day Program, is not supported by the data. Therefore it is necessary to reject the null hypothesis. The statistic $t_2 = 2.21$ is significant at the .05 level. It is interesting to note that all 7 of the full-day children showed a positive gain score, while only 9 of the 13 half-day children

gained positively. The 4 remaining children had a mean negative gain score of 6.5 (see Appendix B).

The final hypothesis $H_o: \psi_3 = 0$) to be examined, i.e., there is no difference in the mean scores observed between the control group attending the half-day session and the control group in the full-day program, is not supported by the data, which again leads one to reject the null hypothesis. Under this hypothesis the test statistic $t_3 = 2.07$ is significant at the .05 level. In this case four of the five full-day children recorded a positive gain score and only eight of the 15 half-day children showed a positive gain score. The mean loss for the seven children with negative gain scores was 2.8 (see Appendix B).

These results are summarized in the following table:

Table 6

SUMMARY OF PLANNED COMPARISON AMONG MEANS

Planned Comparison	Null Hypothesis	t Value	Significance Level
$g_1 + g_2$ vs. $g_3 + g_4$	$\psi_1 = 0$	.524	-
g_1 vs. g_2	$\psi_2 = 0$	2.21	.05
g_3 vs. g_4	$\psi_3 = 0$	2.07	.05

IV

CONCLUSIONS

The results of this study have implications both for actual classroom practice as it applies to the arithmetic curriculum of the elementary school kindergarten and for detailed research pertaining to the number experiences appropriate for kindergarten children.

The results certainly imply that the special instructions used in this experiment do not alone sufficiently enhance the subject's acquisition of the concept of conservation of numerousness. However, it is the feeling of the investigators and the teachers involved in the study that the 12 lessons are an important and useful addition to the present kindergarten curriculum. Work toward a revised and possibly lengthened version of these lessons would appear to be a worthwhile and necessary project.

Also, the test results indicate that further experimentation is needed to perfect the testing instrument. The vocabulary used in the directions to the children appears to be an important factor in the child's understanding of the task he is being asked to perform and therefore it is an important factor in determining the child's response to each question. The

elimination of this vocabulary factor would no doubt lead to a more accurate testing of children's strengths and weaknesses in the basic number concept of conservation of numerousness.

It is clear that in this instance the formal number of experiences given the full-day children in the form of activities suggested in the SRA Teacher's Manual for kindergarten and similar materials, along with original games and materials of the teacher contribute at least as much to the improvement of conservation of numerousness ability as do the special instructions. However, the relationship between the special instructions and the activities involving formal instruction in number experiences, and the contribution of each to the attainment of this concept, is not clear.

The observations of the investigators during the course of this research project along with the statistical results of the study indicate that the 12 30-minute lessons could easily be lengthened and might well become a part of the mathematics curriculum for kindergarten children with a socially deprived background. The results of this study might indicate that the most effective use of the special instructions for socially deprived kindergarten children would be as a supplement to formal activities with number concepts. The implications for children of middle- and low-socioeconomic background are inconclusive.

The investigators recognize the need for further experimentation to determine the most effective techniques in teaching basic number

concepts and for making children aware of the important properties of sets of objects. Therefore, a similar study is planned for the 1970-71 school year. A revised set of lessons will be used on four- and five-year-old kindergarten children at the Latin School of Chicago. Hopefully, an improved testing instrument will be available at that time.

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APPENDIX A

TREATMENT GROUP

Subject	I.Q.	C. Age	Pre-test	Post-test
1	99	5-7	12	12
2	98	5-2	12	14
3	77	6-5	13	4
4	100	5-2	13	6
5	100	5-3	4	8
6	100	5-4	7	11
7	97	5-2	2	10
8	98	5-0	4	13
9	105	5-3	7	3
10	120	5-2	12	14
11	123	5-8	10	13
12	101	5-1	13	7
13	87	4-10	10	13
14	85	6-6	8	14
15	67	5-8	7	14
16	93	5-7	7	13
17	87	5-4	4	11
18	94	5-3	7	13
19	94	4-9	10	11
20	79	5-8	13	14

CONTROL GROUP

Subject	I.Q.	C. Age	Pre-test	Post-test
1	74	4-10	5	10
2	103	5-1	12	13
3	103	5-6	10	13
4	105	5-2	12	14
5	100	4-11	10	8
6	79	5-6	14	11
7	109	5-2	14	12
8	83	5-4	0	1
9	116	5-4	9	12
10	93	5-8	15	10
11	103	5-4	6	8
12	91	5-10	10	8
13	119	5-9	12	7
14	118	5-3	13	14
15	111	5-7	13	12
16	98	5-3	8	14
17	98	5-4	3	9
18	-	-	7	14
19	94	5-0	13	12
20	75	5-7	9	12

APPENDIX B

TABLE OF GAIN SCORES

Treatment		Control	
full-day	half-day	full-day	half-day
g_1	g_2	g_3	g_4
+6	0	+6	+5
+7	+2	+6	+1
+6	-9	+7	+3
+7	-7	-1	+2
+6	+4	+3	-2
+1	+4		-3
+1	+8		- 2
	+9		+1
	-4		+3
	+2		-5
	+3		+2
	-6		-2
	+3		-5
			+1
			-1
mean 4.86	mean .69	mean 4.2	mean -.13
gains 7	gains 9	gains 4	gains 8
loss 0	loss 4	loss 1	loss 7

Treatment	Control
mean 2.05	mean .95
gains 15	gains 12
loss 4	loss 8
mean gain 4.6	mean gain 3.3
mean loss 6.5	mean loss 2.8